A GLOSSARY FOR PHYSICAL DYNAMICS

I gather together here some short definitions and explanations of many of the terms we use to describe frames of reference, rotating bodies, Lagrangian and Hamiltonian mechanics, and fluid mechanics.

- •) CENTRE OF MASS FRAME: A coordinate system with its origin located at the centre of mass of all the particles in the system being studied.
- •) BODY FIXED FRAME: A coordinate system fixed in a rigid body and moving with it. Its origin is usually taken at the centre of mass of the rigid object.
- •) PRINCIPAL AXIS SYSTEM: A coordinate system in which the moment of inertia tensor for a rigid body is diagonal. For any choice of origin there is always at least one principal axis system. Normally we take the origin to be at the centre of mass of the rigid body and use a body fixed frame so that the moment of inertia tensor remains diagonal as the rigid body moves.
- •) PRINCIPAL AXIS: One of the axes in a principal axis system. An axis about which a rigid body may rotate freely (no external torque) with its angular velocity and angular momentum vectors parallel.
- •) PRECESSION: In rotational motion we use precession in at least two different ways. For a rotating rigid body subject to an external force producing a torque, the angular momentum vector $\boldsymbol{L}$ may move in a circle about the axis defined by the direction of the applied force as in the case of gyroscopic precession. For a force and torque-free rotating rigid body the angular velocity vector $\boldsymbol{\omega}$ may rotate about a principal axis as viewed from a body fixed frame of reference. The sense of the term is that there is one vector moving in a bounded path about another vector which may be regarded as fixed in one frame of reference.
- •) NUTATION: The motion of a top with one point fixed (for example a gyroscope) may show simple precession as above. However, a more general motion is possible in which the symmetry axis of the top moves up and down with respect to the direction of gravity as well as precessing around the direction of gravity. It is a "nodding" motion.
- •) LAGRANGIAN: The Lagrangian $L(q_j, \dot{q}_j)$ is the difference of the kinetic and potential energies of a conservative mechanical system expressed in terms of the generalised coordinates q_j and velocities $\dot{q}_j$ which describe the degrees of freedom of the system,

$$L = L(q_j, \dot{q}_j) = T - V \quad .$$

- •) DEGREES OF FREEDOM: A mechanical system is said to have n degrees of freedom if there are n independent ways in which it can move.

- •) GENERALISED COORDINATES: Generalised coordinates for a system with n degrees of freedom are any set of n numbers q_j, $j = 1, 2, \ldots n$, whose values uniquely specify the position or configuration of all the degrees of freedom.
- •) CONFIGURATION SPACE: An n-dimensional space whose coordinates are the generalised coordinates q_j, $j = 1, 2, \ldots, n$. A point in this space fixes uniquely the configuration (position) of all the particles in the system.
- •) LAGRANGE'S EQUATIONS:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \quad , \quad j = 1, 2, \ldots, n \quad .$$

- •) GENERALISED FORCE: In a conservative system with generalised coordinates q_j, the generalised force Q_j associated with coordinate q_j is given by $Q_j = -\frac{\partial V}{\partial q_j}$.
- •) GENERALISED MOMENTUM: The generalised momentum p_j conjugate to the generalised coordinate q_j is defined as $p_j = \frac{\partial L}{\partial \dot{q}_j}$.
- •) CYCLIC COORDINATE: A generalised coordinate q_j is said to be cyclic or ignorable if it does not appear explicitly in the Lagrangian L. The momentum conjugate to a cyclic coordinate is always conserved. The statement that L does not depend on q_j means that the system is symmetric with respect to changes in q_j.
- •) ACTION: The action S is defined with respect to a time interval from time t_1 to time t_2 by

$$S = \int_{t_1}^{t_2} L(q_j(t), \dot{q}_j(t))\, dt \quad ,$$

where $q_j(t)$ is a trajectory from time t_1 to t_2 and L is the Lagrangian function for the system.

- •) HAMILTON'S PRINCIPLE: Amongst all the possible trajectories $q_j(t)$ which take the system from time t_1 to time t_2, the physical trajectory is the one which makes the action an extremum (usually a minimum but could be a saddle point). Formally, $\delta S = 0$ for the actual physical path taken where δS is the variation in the action.
- •) HAMILTONIAN: In the Lagrangian picture the energy is defined by

$$E = \sum_{j=1}^{n} p_j \dot{q}_j - L = T + V \quad .$$

If we solve for the generalised velocities $\dot{q}_i$ in terms of the generalised momenta p_j and then express the energy E as a function of q_j and p_j, we obtain the Hamiltonian

$$H = H(q_j, p_j) = T + V \quad .$$

- •) HAMILTON'S CANONICAL EQUATIONS OF MOTION: In the Hamiltonian description the basic equations of motion are $2n$ first order equations of the form

$$\dot{q}_j = \frac{\partial H}{\partial p_j} \quad , \quad \dot{p}_j = -\frac{\partial H}{\partial q_j} \quad , \quad j = 1, 2, \ldots, n \quad .$$

- •) PHASE SPACE: A $2n$-dimensional space whose coordinates are the q_j, p_j, $j = 1, 2, \ldots, n$. A point in this space fixes uniquely the configuration and all momenta for the system. If we regard these as initial data, then Hamilton's equations uniquely determine the subsequent motion of this phase point which sweeps out a trajectory in phase space.

- •) EULER'S EQUATION FOR AN IDEAL FLUID: The flow of an ideal (non-viscous) incompressible fluid with velocity field $\boldsymbol{v}(\boldsymbol{r})$, pressure field $p(\boldsymbol{r})$, mass density ρ and subject to an external force per unit volume $\boldsymbol{F}(\boldsymbol{r})$, obeys the equation

$$\rho \frac{D\boldsymbol{v}}{Dt} = \rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = \boldsymbol{F} - \boldsymbol{\nabla} p \quad .$$

The condition of incompressibility means that we have also the equation $\boldsymbol{\nabla} \cdot \boldsymbol{v}(\boldsymbol{r}) = 0$.

- •) VORTICITY: The vorticity of a velocity field $\boldsymbol{v}$ is defined as $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$.

- •) IRROTATIONAL FLOW: A velocity field whose vorticity vanishes everywhere, $\boldsymbol{\omega} = 0$.

- •) BERNOULLI'S THEOREM: For the time-independent irrotational flow of an incompressible ideal fluid, subject to a conservative external force $\boldsymbol{F}$, one can prove that the quantity $\rho v^2/2 + V + p$ is constant throughout the fluid where $\boldsymbol{F} = -\boldsymbol{\nabla} V$.