Introduction to Magnetism

Was discovered and found practical application (compass) before construction of a complete theory.
Magnetism is a second order relativistic electrostatic effect (extra force - over and above the electrostatic one - arising from the compression of space ahead of moving charges, and they do move very fast!). The associated force is thus electrostatic in nature and not new type of force.
We have seen that a force is associated with an electric field when acting on a charge. Since there is a force associated with magnetic effects (magnets do attract) we will define the magnetic field B as that type of field that produces a force on a conductor carrying a current. This magnetic field B is measured in *teslas*.

We should remember that this in no way implies generalisation of some electrostatic results into magnetism, e.g. there are no positive or negative magnetic charges (or isolated magnetic poles), magnetic capacitors.

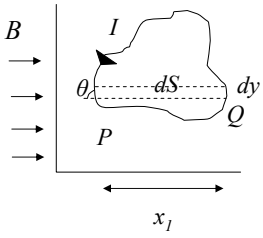
Basic magnetic equations

Lets consider a situation when a field B acts at an angle θ on a short length of conductor dl carrying a current I. It is found that conductor would experience force F associated with B in the direction $\perp$ to the plane containing B and dl.

$$F = BIdl\sin\theta$$

We now going to *simplify* situation a little, so there are no Qs of where this currents are coming from and where they are going to and consider an irregular shaped (again4simplicity) coil interacting with the field. The force (pointing out) will be acting on small element dl at P:

$$F = BIdl\sin\theta = BIdy \quad (dy = dl\sin\theta \text{ from geometry and smallness})$$



But at Q there will be equal force and pointing in opposite direction. These two forces will exert what is called a couple:

$Fx_1 = BIx_1dy = BIdS$

thus the coil behaves as though it had a *magnetic moment IS*

(analogy : $F = qE$, $p = ql \Rightarrow Fl = pE$, but B instead of E in our case)

So what else we are to do but to define magnetic moment

$$m = IS$$

Our coil also behaves as a magnet in the magnetic field

Basic magnetic equations cntd.

Now, in a fashion similar (but not identical!) to electrostatics lets increase the "concentration" of coils, i. e. consider n_0 turns of coil per unit length – a *solenoid*. The solenoid is going to be very long so free ends do not produce any affect in the centre. Now if we place our small test coil in the centre of this solenoid and run current I through, then the solenoid will produce a couple on the coil from which we can deduce that the magnetic field is:

$$B = \mu_0 I n_0$$

Where $\mu_0 = 4\pi * 10^{-7} H/m$ – *permeability of free space*. Suppose the solenoid was of length l and cross section area S and having a uniform magnetic moment M per unit area. The total moment of this solenoid-magnet is then:

$$M * volume = MSl$$

but we know that each turn produces

$$m = IS \Rightarrow MSl = n_0 lm = n_0 lIS \Rightarrow M = In_0 \Rightarrow B = \mu_0 In_0 = \mu_0 M$$

Now lets fill our empty solenoid (either infinite or toroidal to eliminate edge effects) with some material. Guess what:

$$B = \mu\mu_0 In_0$$

Basic magnetic equations cntd.

But we can also regard B as having two parts: one due to solenoid itself ($\mu\mu_0 In_0$) and other due to magnetic moment M of the material ($\mu_0 M$):

$B = \mu\mu_0 In_0 = \mu_0 In_0 + \mu_0 M = \mu_0 (H + M)$, where H is magnetic moment (or magnetisation) of solenoid

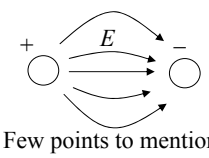
$B = \mu\mu_0 H = \mu_0 H + \mu_0 M$

$\mu - 1 = M / H = \chi$ volume susceptibility of magnetic material

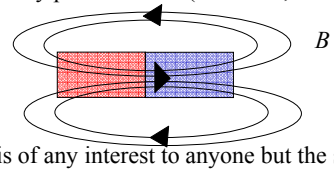
If there are n atomic (molecular, electronic) units per unit volume each of which acquires magnetic moment m then we have:

$$M = nm \Rightarrow \mu - 1 = \frac{nm}{H} = n\alpha$$

where α is defined as magnetic polarisability per unit atom (molecule, electron).



but



Few points to mention. Why all this stuff is of any interest to anyone but the students who have to pass exam. Magnetic fields and living things, magnetic fields and projectiles (no friction – nice!), Magnetic fields and charged plasma (thermonuclear power, thermonuclear fusion).

Magnetic zoo

Magnetic zoo is populated by the following "isms":

(i) Diamagnetism (reaction against the field)

(ii) Paramagnetism (reaction along the field, opposed by thermal effects)

(iii) Ferromagnetism (spontaneous magnetic alignment in fairly weak field) and related antiferromagnetism

(iv) Ferrimagnetism.

Lets look at them in turn.

Diamagnetism – is a very weak form of magnetism that is only exhibited in the presence of an external magnetic field. It is the result of changes in the orbital motion of electrons due to the external magnetic field. The induced magnetic moment is very small and in a direction opposite to that of the applied field.

Diamagnetism is found in all materials; however, because it is so weak it can only be observed in materials that do not exhibit other forms of magnetism. Also, diamagnetism is found in elements with paired electrons. But, oxygen was once thought to be diamagnetic, but a new revised molecular orbital (MO) model confirmed oxygen's paramagnetic nature.

Magnetic zoo cntd.

Paramagnetic materials attract and repel like normal magnets when subject to a magnetic field. This alignment of the atomic dipoles with the magnetic field tends to strengthen it, and is described by a relative magnetic permeability greater than unity (or, equivalently, a small positive magnetic susceptibility).

Paramagnetism requires that the atoms individually have permanent dipole moments even without an applied field, which typically implies a partially filled electron shell. In pure paramagnetism, these atomic dipoles *do not interact* with one another and are randomly oriented in the absence of an external field (due to T effects), resulting in zero net moment.

Now, if there is *interaction* between magnetic dipoles a material can exhibit *ferromagnetism* or *anti-ferromagnetism*. The former is responsible for most of the magnetic behavior encountered in everyday life and, along with ferrimagnetism (magnetic moments on different sublattices in a magnetci material are opposite but unequal), is the basis for all permanent magnets.

In materials that exhibit *antiferromagnetism*, the spins of electrons align in a regular pattern with neighboring spins pointing in opposite directions. This is the opposite of ferromagnetism. Generally, antiferromagnetic materials exhibit antiferromagnetism at a low temperature, and become disordered above a certain temperature; the transition temperature is called the Neel temperature.

The property of ferromagnetism is due to the direct influence of two effects from quantum mechanics: *spin* and the *Pauli exclusion principle*.
The spin of an electron, combined with its orbital angular momentum, results in a magnetic dipole moment and creates a magnetic field. (The classical analogue of quantum-mechanical spin is a spinning ball of charge, but the quantum version has distinct differences, such as the fact that it has discrete up/down states that are not described by a vector; similarly for "orbital" motion, whose classical analogue is a current loop.) In many materials (specifically, those with a filled electron shell), however, the total dipole moment of all the electrons is zero (e.g. the spins are in up/down pairs). Only atoms with partially filled shells (e.g. unpaired spins) can experience a net magnetic moment in the absence of an external field. A ferromagnetic material has many such electrons, and if they are aligned they create a measurable macroscopic field.
These permanent dipoles (often called simply "spins" even though they also generally include orbital angular momentum) tend to align in parallel to an external magnetic field, an effect called paramagnetism. (A related but much smaller effect is diamagnetism, due to the orbital motion induced by an external field, resulting in a dipole moment opposite to the applied field.) Ferromagnetism involves an additional phenomenon, however: the dipoles tend to align spontaneously, without any applied field. This is a purely quantum-mechanical effect.

According to classical electromagnetism, two nearby magnetic dipoles will tend to align in opposite directions (which would create an antiferromagnetic material). In a ferromagnet, however, they tend to align in the same direction because of the Pauli principle: two electrons with the same spin state cannot lie at the same position, and thus feel an effective additional repulsion that lowers their electrostatic energy. This difference in energy is called the exchange energy and induces nearby electrons to align.
At long distances (after many thousands of ions), the exchange energy advantage is overtaken by the classical tendency of dipoles to anti-align. This is why, in an equilibriated (non-magnetized) ferromagnetic material, the dipoles in the whole material are not aligned. Rather, they organize into **magnetic domains** that are aligned (magnetized) at short range, but at long range adjacent domains are anti-aligned. The transition between two domains, where the magnetization flips, is called a *Domain wall* (e.g. a Bloch/Néel wall, depending upon whether the magnetization rotates parallel/perpendicular to the domain interface) and is a gradual transition on the atomic scale (covering a distance of about 300 ions for iron).

Thus, an ordinary piece of iron generally has little or no net magnetic moment. However, if it is placed in a strong enough external magnetic field, the domains will re-orient in parallel with that field, and will remain re-oriented when the field is turned off, thus creating a "permanent" magnet. This magnetization as a function of the external field is described by a *hysteresis* curve. Although this state of aligned domains is not a minimal-energy configuration, it is extremely stable and has been observed to persist for millions of years in seafloor magnetite aligned by the Earth's magnetic field (whose poles can thereby be seen to flip at long intervals). The net magnetization can be destroyed by heating and then cooling (annealing) the material without an external field, however.

As the temperature increases, thermal oscillation, or entropy, competes with the ferromagnetic tendency for dipoles to align. When the temperature rises beyond a certain point, called the **Curie temperature**, there is a second-order phase transition and the system can no longer maintain a spontaneous magnetization, although it still responds paramagnetically to an external field. Below that temperature, there is a spontaneous symmetry breaking and random domains form (in the absence of an external field). The Curie temperature itself is a *critical point*, where the magnetic susceptibility is theoretically infinite and, although there is no net magnetization, domain-like spin correlations fluctuate at all length scales.